Higgs bosons in the simplest SUSY models

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Abstract

Nowadays in the MSSM the moderate values of $\tan\beta$ are almost excluded by LEPII lower bound on the lightest Higgs boson mass. In the Next–to–Minimal Supersymmetric Standard Model the theoretical upper bound on it increases and reaches maximal value in the strong Yukawa coupling limit when all solutions of renormalization group equations are concentrated near the quasi fixed point. For calculation of Higgs boson spectrum the perturbation theory method can be applied. We investigate the particle spectrum in the framework of the modified NMSSM which leads to the self-consistent solution in the strong Yukawa coupling limit. This model allows one to get $m_h \sim 125$ GeV at values of $\tan\beta \geq 1.9$. In the investigated model the lightest Higgs boson mass does not exceed 130.5 ± 3.5 GeV. The upper bound on the lightest CP–even Higgs boson mass in more complicated supersymmetric models is also discussed.

1 Introduction

Last year there was a great progress in the Higgs boson searches. The experimental lower bound on the Higgs boson mass in the Standard Model (SM) has increased from 95.2 GeV [1] to 113.3 GeV [2]. At the same time the upper bound that comes from an analysis of radiative corrections to the electroweak observables has diminished to 210 GeV [2]. Thus the allowed region of the Higgs boson mass in the SM has shrunk drastically. Moreover at the LEPII a few additional $b\bar{b}$ events were observed [3]. They can be interpreted as a signal of the Higgs boson production with mass 115 GeV in the e^+e^- annihilation. Such Higgs boson mass does not agree with theoretical lower bound in the SM that follows from the stability of the physical vacuum up to the Planck scale $M_{Pl} \approx 2.4 \cdot 10^{18}$ GeV [4]–[7].
The simplest supersymmetric (SUSY) extension of SM is the Minimal Supersymmetric Standard Model (MSSM). Its Higgs sector includes two doublets H_1 and H_2. Each of them after spontaneous symmetry breaking acquires a nonzero vacuum expectation value v_1 and v_2 respectively. Instead of them the sum of their squares $v^2 = v_1^2 + v_2^2$ and the value of $\tan\beta = v_2/v_1$ are usually used.
An important feature of supersymmetric models is the existence of a light Higgs boson in the CP–even Higgs sector. The upper bound on its mass strongly depends on the value of $\tan\beta$. At the tree level the lightest Higgs boson mass does not exceed [8] the Z–boson mass: $m_h \le M_Z|\cos 2\beta|$. The loop corrections from the t–quark and its superpartners significantly raise the upper bound on m_h:

$$m_h \le \sqrt{M_Z^2 \cos^2 2\beta + \Delta}\ , \tag{1}$$

where Δ in the one–loop approximation is given by

$$\Delta \approx \frac{3}{2\pi^2}\frac{m_t^4}{v^2}\left[\ln\frac{M_S^2}{m_t^2} + \frac{X_t^2}{M_S^2}\left(1 - \frac{X_t^2}{12M_S^2}\right)\right]. \tag{2}$$

Here m_t is the running top quark mass at the electroweak scale (at $q = M_t^{pole} = 174$ GeV), X_t is the stop mixing parameter and M_S is the SUSY breaking scale which is expressed via the stop masses $m_{\tilde{t}_1}$ and $m_{\tilde{t}_2}$: $M_S = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$. The one–loop corrections (2) attain maximal value at $X_t = \pm\sqrt{6}M_S$. These corrections are proportional to m_t^4 and depend logarithmically on the SUSY breaking scale M_S. They are almost insensitive to the choice of $\tan\beta$. The absolute value of Δ is of order of M_Z^2. The one–loop and two–loop corrections to the lightest Higgs boson mass were

calculated and analysed in [9] and [10] respectively. The upper bound on the lightest Higgs boson mass grows with increasing of $\tan\beta$ and $\ln M_S^2/m_t^2$, and for large $\tan\beta$ ($\tan\beta \gg 1$) reaches $125 \div 128$ GeV. In [6] the bounds on the mass of the Higgs boson in the SM and MSSM were compared.
However the large values of $\tan\beta$ are unpreferable for two reasons. First of them is the proton decay. If one assumes that electroweak and strong interactions are embedded in the SUSY Grand Unified Theory (GUT) at high energies then too fast proton decay is induced due to $d = 5$ operators. The major decay mode is $p \to \bar{\nu} K^+$. The proton life–time τ_p in this case is inversely proportional to $\tan^2\beta$. When $\tan\beta$ is large enough the calculated in the framework of SUSY GUT models proton life–time contradicts experimental restriction on it. Another problem of large $\tan\beta$ scenario concerns flavour–changing neutral currents. The branching ratio $b \to s\gamma$ rises with $\tan\beta$ as $Br(b \to s\gamma) \sim \tan^2\beta$. Thus the values of $\tan\beta \gg 1$ ($\tan\beta \gtrsim 40$) lead to the unacceptable large flavour transitions.
In the case of moderate values of $\tan\beta$ ($\tan\beta \ll 50$) the b–quark and τ–lepton Yukawa couplings are small and one can get an analytical solution of one–loop renormalization group equations [11]. For the top quark Yukawa coupling constant $h_t(t)$ and gauge couplings $g_i(t)$ the analytical solution has the following form:

$$Y_t(t) = \frac{\dfrac{E(t)}{6F(t)}}{\left(1 + \dfrac{1}{6Y_t(0)F(t)}\right)} , \quad \tilde{\alpha}_i(t) = \frac{\tilde{\alpha}_i(0)}{1 + b_i\tilde{\alpha}_i(0)t} , \tag{3}$$

$$E(t) = \left[\frac{\tilde{\alpha}_3(t)}{\tilde{\alpha}_3(0)}\right]^{16/9} \left[\frac{\tilde{\alpha}_2(t)}{\tilde{\alpha}_2(0)}\right]^{-3} \left[\frac{\tilde{\alpha}_1(t)}{\tilde{\alpha}_1(0)}\right]^{-13/99} , \quad F(t) = \int_0^t E(\tau)d\tau ,$$

where $Y_t(t) = \left(\dfrac{h_t(t)}{4\pi}\right)^2$ and $\tilde{\alpha}_i(t) = \left(\dfrac{g_i(t)}{4\pi}\right)^2$. The index i varies from 1 to 3 that corresponds to $U(1)$, $SU(2)$ and $SU(3)$ gauge interactions. The coefficients b_i of one–loop beta functions of $\tilde{\alpha}_i(t)$ are $b_1 = 33/5$, $b_2 = 1$, $b_3 = -3$. The initial conditions $Y_t(0)$ and $\alpha_i(0)$ for the MSSM renormalization group equations are usually set at the Grand Unification scale $M_X \approx 3 \cdot 10^{16}$ where all gauge coupling constants coincide. The variable t is defined by a common way $t = \ln(M_X^2/q^2)$.
Substituting the numerical values of the gauge couplings one finds that at the electroweak scale the second term in the denominator of the expression

describing the evolution of $Y_t(t)$ is approximately equal to $\dfrac{1}{10h_t^2(0)}$. When $h_t^2(0) \geq 1$ the dependence of the top quark Yukawa coupling on its initial value $Y_t(0)$ disappears and all solutions are concentrated near the quasi-fixed point [12]:

$$Y_{QFP}(t_0) = \frac{E(t_0)}{6F(t_0)} \ , \tag{4}$$

where $t_0 = 2\ln(M_X/M_t^{pole}) \approx 65$. Together with $Y_t(t)$ the trilinear scalar coupling of the Higgs boson doublet H_2 with stops $A_t(t)$ and some combination of their masses $\mathfrak{M}_t^2(t) = m_Q^2 + m_U^2 + m_2^2$ are also driven to the infrared quasi–fixed points. In the vicinity of these points $A_t(t)$ is proportional to the universal gaugino mass $M_{1/2}$ at the scale M_X and $\mathfrak{M}_t^2(t) \sim M_{1/2}^2$. Although the solutions of MSSM renormalization group equations achieve quasi–fixed points only for infinite values of $Y_t(0)$, the deviations from them at the electroweak scale are determined by the ratio $\dfrac{1}{6F(t_0)Y_t(0)}$ which is quite small if $h_t^2(0) \geq 1$.

The behaviour of solutions of the MSSM renormalization group equations near the quasi–fixed point at $\tan\beta \sim 1$ and particle spectrum have been studied by many authors [13]–[15]. It has been shown that in the vicinity of this point the $b-\tau$ Yukawa coupling unification is realized [13]. In the recent publications (see [15]–[17]) the value of $\tan\beta$ that corresponds to quasi–fixed point regime has been calculated. It is restricted between 1.3 and 1.8. Such comparatively low values of $\tan\beta$ yield more stringent bound on the lightest Higgs boson mass in the MSSM. The latter does not exceed 94 ± 5 GeV [15],[16]. The obtained theoretical bound on m_h has to be compared with the lower experimental one in the SM since it was computed for the SUSY breaking scale of order of 1 TeV when all other Higgs bosons and superparticles are heavy enough. The straightforward comparison shows that the quasi–fixed point scenario and the considerable part of MSSM parameter space are almost excluded by LEPII data.

Thus the theoretical analysis of the Higgs sector in nonminimal supersymmetric models is stimulated. In this article the spectrum of the Higgs bosons in the Next–to–Minimal Supersymmetric Model is reviewed. The lightest Higgs boson mass in the NMSSM attains its maximum value in the strong Yukawa coupling limit, when the Yukawa couplings are much larger then the gauge ones. All supersymmetric models contain a large number of free parameters what is the main obstacle in the way of their investigations. For example, each SUSY model includes three or four independent

SUSY breaking constants which determine the SUSY particles spectrum. Nevertheless, in the strong Yukawa coupling limit the solutions of the renormalization group equations are focused near the quasi–fixed point which simplifies the analysis. We propose a modification of the NMSSM which allows one to get $m_h \sim 125$ GeV for moderate values of $\tan\beta$ and study Higgs boson spectrum of the model. In the last part the lightest Higgs boson mass in more complicated SUSY models is considered.

2 Higgs sector of NMSSM

2.1 The μ–problem and parameters of NMSSM

The simplest extension of the MSSM is the Next–to–Minimal Supersymmetric Standard Model (NMSSM). Historically NMSSM was suggested as a solution of the μ–problem in the supergravity (SUGRA) models [18]. In addition to observable superfields these models contain a "hidden" sector where local supersymmetry is broken. In the superstring inspired SUGRA models the "hidden" sector always includes the singlet dilaton S and moduli T_m superfields. They appear in the four–dimensional theory as a result of compactification of extra dimensions. The full superpotential of SUGRA models can be presented as expansion in powers of observable superfields

$$W = \hat{W}_0(S, T_m) + \mu(S, T_m)(\hat{H}_1 \hat{H}_2) + h_t(S, T_i)(\hat{Q}\hat{H}_2)\hat{U}^c_R + \dots\ , \tag{5}$$

where $\hat{W}_0(S, T_m)$ is the superpotential of the "hidden" sector. From the expansion (5) it is obvious that parameter μ should be of order of the Planck scale because that is the only scale characterizing the "hidden" (gravity) sector of the theory. On the other hand if $\mu \sim M_{Pl}$ then the Higgs doublets get huge positive masses $m^2_{H_1,H_2} \simeq \mu^2 \simeq M^2_{Pl}$ and electroweak symmetry breaking does not occur at all.

In the NMSSM a new singlet superfield Y is introduced. By definition the superpotential of this model is invariant with respect to the Z_3 discrete transformations [19]. The Z_3 symmetry usually arises in the superstring inspired models in which all observable superfields are massless in the exact supersymmetry limit. The term $\mu(\hat{H}_1\hat{H}_2)$ does not satisfy the last requirement. Therefore it must be eliminated from the NMSSM superpotential. Instead of it the sum of two terms

$$W_h = \lambda \hat{Y}(\hat{H}_1\hat{H}_2) + \frac{\varkappa}{3}\hat{Y}^3\ . \tag{6}$$

arises [18]–[20]. After electroweak symmetry breaking the singlet field Y acquires a nonzero vacuum expectation value ($<Y>= y/\sqrt{2}$) and the effective μ–term ($\mu=\lambda y/\sqrt{2}$) is generated.
The NMSSM superpotential contains a lot of Yukawa couplings. But at the moderate values of $\tan\beta$ all of them are small and can be neglected except for h_t, λ and $\varkappa$. In addition to the Yukawa couplings the lagrangian of the NMSSM contains a large number of soft supersymmetry breaking parameters. Each of the scalar and gaugino fields has a soft mass (m_i and M_i respectively). Each of the Yukawa couplings corresponds to the trilinear scalar coupling A_i in the full lagrangian. The number of these unknown parameters can be considerably reduced if one assumes the universality of the soft SUSY breaking terms at the scale M_X. Then only three independent dimensional parameters are left: the universal gaugino mass $M_{1/2}$, the universal scalar mass m_0 and the universal trilinear coupling of scalar fields A. Naturally universal soft SUSY breaking terms appear in the minimal supergravity model [21] and in the simplest models deduced from the superstring theories [22]. The universal parameters of soft supersymmetry breaking determined at the Grand Unification scale have to be considered as boundary conditions for the renormalization group equations that describe the evolution of all fundamental constants up to electroweak scale or SUSY breaking scale. The complete system of NMSSM renormalization group equations can be found in [23],[24].

2.2 The CP–even Higgs boson spectrum

The Higgs sector of the Next-to-Minimal Supersymmetric Standard Model includes six massive states. Three of them are CP–even fields, two are CP–odd fields and one is a charged field. The determinants of the mass matrices of the CP–odd and charged Higgs bosons go to zero. It corresponds to the appearance of two Goldstone bosons:

$$\begin{aligned}\eta^0 &= \sqrt{2}\sin\beta\,\mathrm{Im}H_2^0+\sqrt{2}\cos\beta\,\mathrm{Im}H_1^0\,,\\ \eta^+ &= \sin\beta\,H_2^+ +\cos\beta\,(H_1^-)^*\,.\end{aligned}\tag{7}$$

which are swallowed up by the the massive vector $W^{\pm}$ and Z–bosons during the spontaneous breaking of $SU(2)\times U(1)$ symmetry. For this reason the masses of neutral CP–odd bosons and charged boson are easily calculated. In the CP–even Higgs sector the situation is more complex. The CP–even states arise as a result of mixing of the real parts of the neutral components of the Higgs doublets with real part of the field Y. The determinant of their mass matrix does not vanish and in order to calculate its eigenvalues one has to diagonalize the (3×3) mass matrix. Instead of $\mathrm{Re}H_1^0$, $\mathrm{Re}H_2^0$ and $\mathrm{Re}Y$ it is more convenient to consider their linear combinations:

$$
\begin{aligned}
\chi_1 &= \sqrt{2}\cos\beta\,\mathrm{Re}H_1^0 + \sqrt{2}\sin\beta\,\mathrm{Re}H_2^0\,, \\
\chi_2 &= -\sqrt{2}\sin\beta\,\mathrm{Re}H_1^0 + \sqrt{2}\cos\beta\,\mathrm{Re}H_2^0\,, \\
\chi_3 &= \sqrt{2}\,\mathrm{Re}Y.
\end{aligned}
\tag{8}
$$

In the basis (8) the mass matrix of CP–even Higgs fields can be simply written as (see [25])

$$
M^2 = \begin{pmatrix}
\dfrac{\partial^2 V}{\partial v^2} & \dfrac{1}{v}\dfrac{\partial^2 V}{\partial v \partial \beta} & \dfrac{\partial^2 V}{\partial v \partial y} \\[2ex]
\dfrac{1}{v}\dfrac{\partial^2 V}{\partial v \partial \beta} & \dfrac{1}{v^2}\dfrac{\partial^2 V}{\partial \beta^2} & \dfrac{1}{v}\dfrac{\partial^2 V}{\partial y \partial \beta} \\[2ex]
\dfrac{\partial^2 V}{\partial v \partial y} & \dfrac{1}{v}\dfrac{\partial^2 V}{\partial y \partial \beta} & \dfrac{\partial^2 V}{\partial y^2}
\end{pmatrix},
\tag{9}
$$

where $V(v_1, v_2, y)$ is the effective potential of the NMSSM Higgs sector.

It is well known that *the minimum eigenvalue of a matrix does not exceed its minimum diagonal element.* Thus the lightest CP–even Higgs boson mass are always smaller than

$$
m_h^2 \le M_{11}^2 = \frac{\partial^2 V}{\partial v^2} = \frac{\lambda^2}{2} v^2 \sin^2 2\beta + M_Z^2 \cos^2 2\beta + \Delta\,.
\tag{10}
$$

In the right side of inequality (10) Δ is the contribution of loop corrections to the Higgs boson potential. The expression (10) was obtained in the tree level approximation ($\Delta = 0$) in [20]. The contribution of loop corrections to the upper bound on the lightest Higgs boson mass in the NMSSM is almost the same as in the minimal SUSY model. In particular, in order to calculate the corrections from the t–quark and its superpartners one has to replace the parameter μ in the corresponding formulas of MSSM by $\lambda y/\sqrt{2}$. The Higgs boson sector of NMSSM and loop corrections to it were studied in [24]–[26]. In [7] the upper bound on m_h in the NMSSM was compared with theoretical bounds in the SM and in its minimal supersymmetric extension.

The calculation of CP–even Higgs spectrum is simplified in the most realistic case when all superparticles are heavy ($M_S \gg M_Z$). In this case the contributions of new particles to the electroweak observables are suppressed as $\left(\dfrac{M_Z}{M_S}\right)^2$ (see for example [27]). On the other hand the prediction for the values of strong coupling constant at the electroweak scale $\alpha_3(M_Z)$ that can be obtained from the gauge coupling unification [28] is improved with increasing of supersymmetry breaking scale M_S. For $M_S \simeq 1$ TeV it becomes close to the $\alpha_3(M_Z) = 0.118(3)$ which has been found independently from the analysis of the experimental data [29]. Also it should be noted that the lightest Higgs boson mass reaches its maximal value in the SUSY models for $M_S \sim 1 \div 3$ TeV.

In the considered limit the mass matrix (9) has a hierarchical structure and can be represented as a sum of two matrices [25]:

$$M_{ij}^2 = \begin{pmatrix} E_1^2 & 0 & 0 \\ 0 & E_2^2 & 0 \\ 0 & 0 & E_3^2 \end{pmatrix} + \begin{pmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & V_{23} \\ V_{31} & V_{32} & V_{33} \end{pmatrix} . \tag{11}$$

The first matrix is diagonal with $E_1^2 = 0$ and $E_{2,3}^2 \sim M_S^2$. The matrix elements V_{11}, V_{22}, V_{33}, $V_{12} = V_{21}$ are of order of M_Z^2. The other matrix elements that corresponds to mixing of χ_1 and χ_2 with χ_3 are equal

$$V_{13} = V_{31} = \lambda v X_1 \ , \quad V_{23} = V_{32} = \lambda v X_2 \ , \tag{12}$$

where $X_1 \sim X_2 \sim M_S$.

Regarding the ratio $\dfrac{M_Z^2}{E_{2,3}^2}$ as a small parameter the mass matrix (11) can be diagonalized by means of usual quantum mechanical perturbation theory. For the Higgs boson masses it yields:

$$\begin{aligned} m_S^2 &\approx E_3^2 + V_{33} + \lambda^2 v^2 \frac{X_1^2}{E_3^2} + \lambda^2 v^2 \frac{X_2^2}{E_3^2 - E_2^2} \ , \\ m_H^2 &\approx E_2^2 + V_{22} + \lambda^2 v^2 \frac{X_2^2}{E_2^2 - E_3^2} \ , \\ m_h^2 &\approx \frac{\lambda^2}{2} v^2 \sin^2 2\beta + M_Z^2 \cos^2 2\beta + \Delta - \lambda^2 v^2 \frac{X_1^2}{E_3^2} \ . \end{aligned} \tag{13}$$

The explicit expressions for the E_i^2 and V_{ij} can be found in [25]. For the simplicity we restrict our consideration to the first order of perturbation theory and neglect matrix element V_{12} because its contribution to m_i^2 is of order of $\dfrac{M_Z^4}{M_S^2}$.

The perturbation theory becomes inapplicable when $|E_2^2 - E_3^2| \sim \lambda v X_2$. However the mass matrix (11) can be easily diagonalized even in this case. In order to do this one should choose the basis where the matrix element

M_{23} is zero. After that in the new basis the Higgs boson masses can be computed using ordinary results of perturbation theory.
The first three terms in the last relation in (13) reproduce the upper bound on the lightest Higgs boson mass in the NMSSM. Their sum is equal to V_{11} in our notations. The last term in this expression gives a negative contribution to m_h. Even when the ratio $\dfrac{M_Z^2}{M_S^2}$ goes to zero it does not vanish. Thus in the NMSSM the mass of the lightest CP–even Higgs boson can be considerably less than its upper bound [25].

2.3 Renormalization of the Yukawa couplings and soft SUSY breaking terms

According to inequality (10) the upper bound on m_h rises when λ increases and the value of $\tan\beta$ diminishes. For $\tan\beta \gg 1$ the value of $\sin 2\beta$ goes to zero and the upper bound on the lightest Higgs boson mass in the NMSSM coinsides with that one in the minimal SUSY model. With decreasing of $\tan\beta$ the top quark Yukawa coupling at the electroweak scale $h_t(t_0)$ grows. The analysis of the solutions of NMSSM renormalization group equations reveals that the values of the Yukawa couplings at the Grand Unification scale tends to rise with increasing of them at the electroweak scale. As a result the upper bound on the lightest Higgs boson mass in the NMSSM reaches its maximum value in the strong Yukawa coupling limit when the Yukawa couplings are much larger than the gauge ones at the scale M_X.
The renormalization of NMSSM coupling constants in the strong Yukawa coupling limit has been studied in [30],[31]. In the considered case the Yukawa couplings are attracted towards a Hill type effective (quasi–fixed) line ($\varkappa = 0$) or surface ($\varkappa \neq 0$) which restrict the allowed region of h_t, λ and $\varkappa$. Outside this range the solutions of renormalization group equations blow up before the Grand Unification scale M_X and perturbation theory is not valid at $q^2 \sim M_X^2$. While the values of the Yukawa couplings at the scale M_X grow, the region, where all solutions are concentrated, shrinks and $h_t^2(0)$, $\lambda^2(0)$, $\varkappa^2(0)$ are focused near the quasi–fixed points [30]. These points appear as a result of intersection of the quasi–fixed line or surface with the invariant (fixed) line. The latter connects the stable fixed point in the strong Yukawa coupling regime [32] with infrared fixed point of NMSSM renormalization group equations [33]. The properties of invariant lines and surfaces were reviewed in detail in [5], [34].
When the Yukawa couplings tend to quasi–fixed points the trilinear scalar

couplings $A_i(t)$ and some combinations of scalar particle masses $\mathfrak{M}_i^2(t)$, where

$$\begin{array}{rcl} \mathfrak{M}_t^2(t) & = & m_2^2(t) + m_Q^2(t) + m_U^2(t) \ , \\ \mathfrak{M}_\lambda^2(t) & = & m_1^2(t) + m_2^2(t) + m_y^2(t) \ , \\ \mathfrak{M}_\varkappa^2(t) & = & 3m_y^2(t) \ , \end{array} \tag{14}$$

become insensitive to their initial values A and $3m_0^2$ at the scale M_X [31]. For the universal boundary conditions one has

$$\begin{array}{c} A_i(t) = e_i(t)A + f_i(t)M_{1/2} \ , \\ m_i^2(t) = a_i(t)m_0^2 + b_i(t)M_{1/2}^2 + c_i(t)AM_{1/2} + d_i(t)A^2. \end{array} \tag{15}$$

The functions $e_i(t)$, $f_i(t)$, $a_i(t)$, $b_i(t)$, $c_i(t)$ and $d_i(t)$ remain unknown since an analytical solution of NMSSM renormalization group equations hasn't been found yet. While the Yukawa couplings tend to infinity the values of functions $e_i(t_0)$, $c_i(t_0)$ and $d_i(t_0)$ vanish. It means the solutions of renormalization group equations go to the quasi–fixed points too. In the vicinity of the quasi–fixed points $A_i(t)$ are proportional to $M_{1/2}$ and $\mathfrak{M}_i^2(t) \sim M_{1/2}^2$.

3 Particle spectrum in the modified NMSSM

3.1 The modified NMSSM

The fundamental parameters of NMSSM at the Grand Unification scale have to be adjusted so that the minimization conditions of the effective Higgs boson potential are satisfied:

$$\frac{\partial V(v_1, v_2, y)}{\partial v_1} = 0 \ , \quad \frac{\partial V(v_1, v_2, y)}{\partial v_2} = 0 \ , \quad \frac{\partial V(v_1, v_2, y)}{\partial y} = 0 \ . \tag{16}$$

Since the vacuum expectation value v is known they can be used for the calculation of A, m_0 and $M_{1/2}$. But in the strong Yukawa coupling limit it is impossible to get the real solution of nonlinear algebraic equations (16). Thus although the recent investigations [35],[36] reveal that the upper bound on the lightest Higgs boson mass in the NMSSM is larger than the one in the MSSM by $7 \div 10$ GeV, in the considered region of the NMSSM parameter space the self–consistent solution can not be obtained. Such solution of equations (16) appears for $\lambda(0)^2, \varkappa^2(0) \leq 0.1$ when the upper bound on m_h is the same as in the minimal supersymmetric model.
Moreover due to the Z_3 symmetry three degenerate vacuum configurations arise. After a phase transition at the electroweak scale the Universe is

filled by three degenerate phases. The regions with different phases are separated from each other by domain walls. The cosmological observations do not confirm the existence of the domain walls. The domain structure of the vacuum is destroyed if the discrete Z_3 symmetry of NMSSM lagrangian disappears. An attempt of Z_3 symmetry breaking by means of operators of dimension $d = 5$ was made in [37]. It was shown that their introduction leads to quadratic divergences in the two–loop approximation, i.e. to the hierarchy problem. As a result the vacuum expectation value of Y turns out of the order of 10^{11} GeV.

In order to avoid the domain wall problem and get the self–consistent solution in the strong Yukawa coupling limit one has to modify the NMSSM. The simplest way is to introduce the bilinear terms $\mu(\hat{H}_1\hat{H}_2)$ and $\mu'\hat{Y}^2$ in the superpotential which are not forbidden by electroweak symmetry. At the same time one can omit the coupling $\varkappa$, that allows to simplify the analysis of the modified NMSSM. In this case ($\varkappa = 0$) the upper bound on the lightest Higgs boson mass reaches its maximum value. Neglecting all the Yukawa constants except for h_t and λ one gets the following expression for the modified NMSSM (MNSSM) superpotential [38]

$$W_{MNSSM} = \mu(\hat{H}_1\hat{H}_2) + \mu'\hat{Y}^2 + \lambda\hat{Y}(\hat{H}_1\hat{H}_2) + h_t(\hat{H}_2\hat{Q})\hat{U}^C_R \ . \qquad (17)$$

The bilinear terms in the superpotential (17) break the Z_3 symmetry and the domain walls do not arise, because the degenerated vacua do not exist. The introduction of the parameter μ permits to obtain the self–consistent solution of algebraic equations (16) when $h_t^2(0), \lambda^2(0) \gg g_i^2(0)$. In the supergravity models the bilinear terms may be generated due to the additional terms $[Z(H_1H_2) + Z'Y^2 + h.c.]$ in the Kähller potential [39],[40] or due to nonrenormalizable interaction of the Higgs doublet superfields with the "hidden" sector ones [40],[41].

The effective Higgs boson potential of MNSSM can be written as the sum

$$
\begin{aligned}
V(H_1, H_2, Y) = \mu_1^2|H_1|^2 + \mu_2^2|H_2|^2 + \mu_y^2|Y|^2 + \Bigg[\mu_3^2(H_1H_2) + \mu_4^2 Y^2 + \\
+\lambda A_\lambda Y(H_1H_2) + \lambda\mu' Y^*(H_1H_2) + \lambda\mu Y\left(|H_1|^2 + |H_2|^2\right) + h.c.\Bigg] + \\
+\lambda^2|(H_1H_2)|^2 + \lambda^2 Y^2\left(|H_1|^2 + |H_2|^2\right) + \frac{g'^2}{8}\left(|H_2|^2 - |H_1|^2\right)^2 + \\
+\frac{g^2}{8}\left(H_1^+\sigma_a H_1 + H_2^+\sigma_a H_2\right)^2 + \Delta V(H_1, H_2, Y)\,,
\end{aligned}
\tag{18}
$$

where $\Delta V(H_1, H_2, Y)$ is the sum of one–loop corrections to the effective potential, g and g' are the gauge constants of the $SU(2)$ and $U(1)$ interactions respectively ($g_1 = \sqrt{5/3}\,g'$). The parameters μ_i^2 are expressed via the soft SUSY breaking terms as follows

$$
\mu_1^2 = m_1^2 + \mu^2\,, \qquad \mu_2^2 = m_2^2 + \mu^2\,, \qquad \mu_y^2 = m_y^2 + \mu'^2\,,
$$
$$
\mu_3^2 = B\mu\,, \quad \mu_4^2 = \frac{1}{2}B'\mu'\,.
$$

The μ–terms in the superpotential (17) lead to the appearance of bilinear scalar couplings B and B' in the effective potential of the Higgs fields. They arise as a result of the soft supersymmetry breaking. The values of B and B' depend on the mechanism of the μ and μ' generation. For a minimal choice of the fundamental parameters all soft scalar masses and all bilinear scalar couplings should be put equal at the scale M_X

$$
m_1^2(M_X) = m_2^2(M_X) = m_y^2(M_X) = m_0^2\,,
$$
$$
B(M_X) = B'(M_X) = B_0\,.
$$

Thus in addition to the constants of the SM the modified NMSSM contains seven independent parameters

$$
\lambda\ ,\quad \mu\ ,\quad \mu'\ ,\quad A\ ,\quad B_0\ ,\quad m_0^2\ ,\quad M_{1/2}\ .
$$

3.2 The procedure of the analysis

Although the parameter space of the considered model is enlarged it is possible to get some predictions for the Higgs boson spectrum in the strong

Yukawa coupling limit. It is reasonable to start our analysis from the quasi–fixed point of MNSSM [30]:

$$\begin{array}{lll} \rho_t^{QFP}(t_0) = 0.803\ , & \rho_{A_t}^{QFP}(t_0) = 1.77\ , & \rho_{\mathfrak{M}_t^2}^{QFP}(t_0) = 6.09\ , \\ \rho_\lambda^{QFP}(t_0) = 0.224\ , & \rho_{A_\lambda}^{QFP}(t_0) = -0.42\ , & \rho_{\mathfrak{M}_\lambda^2}^{QFP}(t_0) = -2.28\ , \end{array} \tag{19}$$

because all solutions of renormalization group equations are concentrated in its vicinity if $h_t^2(0), \lambda^2(0) \gg g_i^2(0)$. In the relations (19) the following notations are used: $\rho_{t,\lambda}(t) = \dfrac{Y_{t,\lambda}(t)}{\tilde{\alpha}_3(t)}$, $Y_t = \dfrac{h_t^2(t)}{(4\pi)^2}$, $Y_\lambda(t) = \dfrac{\lambda^2(t)}{(4\pi)^2}$, $\rho_{A_{t,\lambda}} = \dfrac{A_{t,\lambda}}{M_{1/2}}$ and $\rho_{\mathfrak{M}^2_{t,\lambda}} = \dfrac{\mathfrak{M}^2_{t,\lambda}}{M^2_{1/2}}$.

For given set of the Yukawa couplings (19) the value of $\tan\beta$ can be extracted from the expression which relates the running t–quark mass $m_t(M_t^{pole})$ with $h_t(t_0)$

$$m_t(M_t^{pole}) = \frac{1}{\sqrt{2}} h_t(M_t^{pole}) v \sin\beta\ . \tag{20}$$

We substitute in the left side of equation (20) the value of the running top quark mass $m_t(M_t^{pole}) = 165 \pm 5\,\mathrm{GeV}$ calculating in the $\overline{MS}$ scheme [42]. The uncertainty in the $m_t(M_t^{pole})$ is determined by the experimental error with which the top quark pole mass is measured $M_t^{pole} = 174.3 \pm 5.1\,\mathrm{GeV}$ [43]. In the infrared quasi–fixed point regime, that corresponds to $h_t^2(0) = \lambda^2(0) = 10$, we obtain $\tan\beta \approx 1.88$ for $m_t(M_t^{pole}) = 165\,\mathrm{GeV}$ [38].

At the first step of our analysis the supersymmetry breaking scale is also fixed by means of the condition $M_3(1000\,\mathrm{GeV}) = 1000\,\mathrm{GeV}$, where M_3 is the gluino mass. The last condition permits to calculate immediately the universal gaugino mass at the Grand Unification scale. It ensures that the superparticles are much heavier than the observable ones.

Next we use the equations (16) that define the minimum of the Higgs boson potential of the modified NMSSM to restrict the allowed region of the parameter space. Instead of μ it is more convenient to introduce

$\mu_{eff} = \mu + \lambda y/\sqrt{2}$. Then after some transformations we obtain

$$\left\{ \begin{array}{l} \mu_{eff}^2 = \dfrac{m_1^2 - m_2^2 \tan^2\beta + \Delta_Z}{\tan^2\beta - 1} - \frac{1}{2} M_Z^2 \\ \sin 2\beta = \dfrac{-2\left(B\mu + \dfrac{\lambda y X_2}{\cos 2\beta}\right)}{m_1^2 + m_2^2 + 2\mu_{eff}^2 + \dfrac{\lambda^2}{2}v^2 + \Delta_\beta} \\ y\left(m_y^2 + \mu'^2 + B'\mu'\right) = \frac{\lambda}{2} v^2 X_1 - \Delta_y \, , \end{array} \right. \tag{21}$$

where Δ_i is the contribution of loop corrections and

$$X_1 = \frac{1}{\sqrt{2}}\left(2\mu_{eff} + (\mu' + A_\lambda)\sin 2\beta\right), \quad X_2 = \frac{1}{\sqrt{2}}(\mu' + A_\lambda)\cos 2\beta.$$

As the values v and $\tan\beta$ are known from the equations (21) one can find the vacuum expectation value y and parameters B_0 and μ_{eff}. In the numerical analysis we take into account only the loop corrections from t–quark and its superpartners, because they give a leading contribution. Therefore Δ_i are the functions of μ_{eff} and do not depend on B_0 and y. From the first equation of (21) the absolute value of μ_{eff} is calculated. The sign of μ_{eff} is not determined and should be considered as a free parameter. The bilinear scalar coupling B_0 and vacuum expectation value y are computed from the two other equations of (21). The last of them points out that the value of y is of the order of $\lambda v^2/M_S$ and much smaller than v if the superparticles are heavy enough.

Since μ_{eff}, B_0 and y have been found we investigate the dependence of the Higgs boson spectrum on A, m_0 and μ' using the relations (15). For the masses of CP–odd states one can get an exact analytical result:

$$m_{A_1,A_2}^2 = \frac{1}{2}\left(m_A^2 + m_B^2 \pm \sqrt{\left(m_A^2 - m_B^2\right)^2 + 4\left(\frac{\lambda v}{\sqrt{2}}(\mu' + A_\lambda) + \Delta_0\right)^2}\,\right), \tag{22}$$

$$m_A^2 = m_1^2 + m_2^2 + 2\mu_{eff}^2 + \frac{\lambda^2}{2}v^2 + \Delta_A,$$

$$m_B^2 = m_y^2 + \mu'^2 - B'\mu' + \frac{\lambda^2}{2}v^2 + \Delta_3.$$

The mass matrix of CP–even Higgs sector has a hierarchical structure and can be written in the form (11) with

$$\begin{aligned}
&E_2^2 = m_1^2 + m_2^2 + 2\mu_{eff}^2\,, \qquad E_3^2 = m_y^2 + \mu'^2 + B'\mu'\,,\\
&V_{11} = M_Z^2\cos^2 2\beta + \frac{1}{2}\lambda^2 v^2\sin^2 2\beta + \Delta_{11}\,,\\
&V_{12} = V_{21} = \left(\frac{1}{4}\lambda^2 v^2 - \frac{1}{2}M_Z^2\right)\sin 4\beta + \Delta_{12}\,,\\
&V_{22} = M_Z^2\sin^2 2\beta + \frac{1}{2}\lambda^2 v^2\cos^2 2\beta + \Delta_A + \Delta_{22}\,,\\
&V_{13} = V_{31} = \lambda v X_1 + \Delta_{13}\,, \qquad V_{23} = V_{32} = \lambda v X_2 + \Delta_{23}\,,\\
&V_{33} = \frac{1}{2}\lambda^2 v^2 + \Delta_{33}\,.
\end{aligned} \tag{23}$$

In the formulas (22) and (23) $\Delta_0, \Delta_3, \Delta_A$ and Δ_{ij} ($\Delta_{11} = \Delta$) are loop corrections to the CP–odd and CP–even mass matrices. The mass matrix of CP–even Higgs sector can be diagonalized and the expressions for the masses of CP–odd states (22) can be simplified using the perturbation theory of quantum mechanics. In the main order of perturbation theory the masses of the heavy Higgs bosons are: $m_H^2 \approx E_2^2$, $m_S^2 \approx E_3^2$, $m_{A_1}^2 \approx m_B^2$ and $m_{A_2}^2 \approx m_A^2$.

3.3 Numerical results

The results of the numerical analysis of the particle spectrum near the MNSSM quasi–fixed point are presented in Figs.1–3. There are two regions of the MNSSM parameter space. In one of them the mass of the lightest CP–even Higgs boson is larger than the upper bound on m_h in the MSSM (see Figs. 1a and 2a) whereas in the other region it is smaller (see Figs. 1b and 2b). As follows from the relations (13) the lightest Higgs boson mass in the NMSSM and in its modification attains its upper bound when V_{13} (or X_1) goes to zero. In the MNSSM it happens if

$$\mu' = -\frac{2\mu_{eff}}{\sin 2\beta} - A_\lambda - \frac{\sqrt{2}\Delta_{13}}{\lambda v\sin 2\beta}\,. \tag{24}$$

Thus m_h is larger in that part of the parameter space where the signs of μ and μ' are opposite. If μ' tends to infinity then the the singlet CP–odd and CP–even fields get huge masses and their contribution to the effective potential of the Higgs bosons vanishes due to the decoupling property. In the considered limit the lightest Higgs boson mass is the same as in the minimal SUSY model.

It is necessary to emphasize that the one–loop and even two–loop corrections give an appreciable contribution to the mass of the lightest CP–even

Higgs boson. So the two–loop corrections [10] reduce its mass approximately by 10 GeV. They nearly compensate the growth of lightest Higgs boson mass with increasing of SUSY breaking scale M_S which arises because of one–loop corrections. Due to loop corrections the values of m_h for $\mu_{eff} > 0$ and $\mu_{eff} < 0$ become different. The contribution of loop corrections Δ rises as the stop mixing parameter $X_t = A_t + \mu_{eff}/\tan\beta$ increases. Since $A_t < 0$ the absolute value of X_t is larger if $\mu_{eff} < 0$.
As the part of the parameter space where μ_{eff} and μ' have the same signs is almost excluded by LEPII data, we investigate the particle spectrum in the case when the signs of μ_{eff} and μ' are opposite. In the most interesting region, where the lightest Higgs boson mass is close to its upper bound, the value of μ' is considerably larger than μ_{eff} and M_S (see (24)). Moreover the product $B'\mu'$ is positive. Indeed from the second relation of the system (21), which defines the minimum of the MNSSM Higgs boson potential (18), it follows that the bilinear scalar coupling B and μ have different signs. As a consequence near the maximum of the curves in Figs. 1a and 2a. the sign of B' coincides with the sign of μ'.
It means, that the heaviest particle in the modified NMSSM is the CP–even Higgs boson that corresponds to the neutral field Y. Its mass is $m_S^2 > \mu'^2$ and is substantially larger than the scale of supersymmetry breaking. As one can see from Figs. 1c and 2c the mass of the other heavy CP–even Higgs boson m_H is almost insensitive to the value of μ' since $m_S^2 \gg m_H^2$. The masses of CP–odd states are always smaller than μ'^2. If the value of μ' grows the mass of the heaviest CP–odd state $m_{A_1}^2$ increases too as $m_{A_1}^2 \sim \mu'^2$. When the value of μ' diminishes the lightest CP–odd boson mass m_{A_2} decreases and for $\mu' \sim B'$ becomes of the order of electroweak scale. At given values of μ' the low constraint on μ' appears which comes from the requirement $m_{A_2}^2 > 0$. However even if the mass of the lightest CP–odd state is of the order of M_Z, it will be quite difficult to observe it in future experiments because the main contribution to its wave function gives the CP–odd component of the singlet field Y. The heaviest fermion in the modified NMSSM is the neutralino ($m_{\tilde{\chi}_5}$) which is a superpartner of the scalar field Y. Its mass is proportional to μ'. The remaining masses of neutralinos ($m_{\tilde{\chi}_i}$), charginos ($m_{\tilde{\chi}_i^{\pm}}$), squarks and sleptons do not depend on the μ'.
The spectrum of new fermion states, squarks and sleptons is also insensitive to the choice of the parameter A, since near the quasi–fixed point (19) the dependence of scalar masses m_i^2 and trilinear scalar couplings A_i on it disappears. For this reason the lightest CP–even Higgs boson mass is

almost independent of A. Nevertheless the dependence of the heavy Higgs boson spectrum on the universal trilinear scalar coupling A is conserved. It occurs because the bilinear scalar coupling B' is proportional to A. The dependence of the Higgs boson masses on the parameter A for $m_0 = 0$ are presented in Figs. 1d and 2d.

Although everywhere in Figs. 1 and 2 we put $m_0 = 0$, the qualitative pattern of the particle spectrum does not change if the universal soft scalar mass varies from zero to $M_{1/2}^2$. It should be noted that the masses of squarks, sleptons, heavy Higgs bosons, heavy charginos and neutralinos rise with increasing of m_0 while the spectrum of the lightest particles remains unchanged.

Up to now the particle spectrum in the quasi–fixed point regime which corresponds to the initial values of the Yukawa couplings $h_t^2(0) = \lambda^2(0) = 10$ has been studied. In the vicinity of the quasi–fixed point (19) for $m_t(M_t^{pole}) = 165\,\mathrm{GeV}$ and $M_3 \le 2\,\mathrm{TeV}$ the lightest Higgs boson mass does not exceed 127 GeV. The results presented in Table 1 point out that the qualitative pattern or MNSSM particle spectrum does not change even if $h_t^2(0) \gg \lambda^2(0)$ or $h_t^2(0) \ll \lambda^2(0)$ as long as the Yukawa couplings are much larger than the gauge ones at the scale M_X. The masses of the Higgs bosons and the masses of the superpartners of the observable particles were calculated there for the values of μ' computed by the formula (24) and $A = m_0 = 0$. At the same time from the Table 1 one can see that the numerical value of the lightest Higgs boson mass is raised from $105 \div 113\,\mathrm{GeV}$ for $\lambda^2(0) = 2$ to $118 \div 128\,\mathrm{GeV}$ for $\lambda^2(0) = 10$.

Therefore at the last stage of our analysis we investigate the dependence of the upper bound on m_h on the choice of the Yukawa couplings at the Grand Unification scale. For each $h_t^2(t_0)$ we find the value of $\tan\beta$ using the relation (20) and choose $\lambda^2(t_0)$ and μ' so that the lightest Higgs boson mass attains its upper bound. The obtained curve $m_h(\tan\beta)$ is plotted in Fig. 3. The upper bound on m_h in the MSSM as a function of $\tan\beta$ is also presented in this Fig. Two bounds are very close for large $\tan\beta$ ($\tan\beta \gg 1$). The curve $m_h(\tan\beta)$ in the MNSSM reaches its maximum when $\tan\beta = 2.2 \div 2.4$ which corresponds to the strong Yukawa coupling limit. The numerical analysis reveals that the mass of the lightest CP–even Higgs boson in the modified NMSSM is always smaller than $130.5 \pm 3.5\,\mathrm{GeV}$, where the uncertainty is mainly explained by the error in the top quark pole mass.

4 Final remarks and conclusions

We have argued that the upper bound on the lightest Higgs boson mass in the NMSSM attains its maximal value in the strong Yukawa coupling limit. In the considered limit all solutions of renormalization group equations gathered near the quasi–fixed points. If the scale of supersymmetry breaking is much larger than the electroweak one the perturbation theory can be used for the calculation of the Higgs boson masses. However even when $M_S \gg M_Z$ the lightest CP–even Higgs boson mass in the NMSSM is appreciably smaller than its upper bound in the dominant part of the parameter space. Besides in the strong Yukawa coupling limit within the NMSSM with a minimal set of fundamental parameters the self–consistent solution does not exist. Moreover the Z_3 symmetry of the NMSSM superpotential leads to the domain wall problem.
We have suggested such modification of the NMSSM that allows to get the self–consistent solution in the strong Yukawa coupling limit and at the same time to avoid the domain wall problem. The superpotential of the modified NMSSM (MNSSM) includes the bilinear terms which break the Z_3 symmetry. We have studied the spectrum of the Higgs bosons in the MNSSM. The qualitative pattern of the particle spectrum is most sensitive to the choice of two parameters — μ' and M_S. The limit $\mu' \gg M_S$, when the CP–even and CP–odd scalar fields become very heavy, corresponds to the minimal SUSY model. In the most interesting region of the parameter space, where the mass of the lightest Higgs boson is larger than one in the MSSM, the Higgs boson mass matrix has the hierarchical structure and can be diagonalized using the method of perturbation theory. The heaviest particle in this region of the MNSSM parameters is the CP–even Higgs boson corresponding to the neutral scalar field Y and the heaviest fermion is $\tilde{Y}$ which is the superpartner of the singlet field Y. The lightest Higgs boson mass in the considered model may reach 127 GeV even for the comparatively low value of $\tan\beta \simeq 1.9$ and does not exceed 130.5 ± 3.5 GeV. The obtained upper bound on the mass of the lightest CP–even Higgs boson is not an absolute one in the supersymmetric models. For instance, the upper bound on m_h increases if new $5 + \bar{5}$ supermultiplets appear in the NMSSM at the SUSY breaking scale. These multiplets change the evolution of gauge couplings. Their values at the intermediate scale rise if a number of new supermultiplets increases. For this reason the allowed region of the Yukawa couplings at the electroweak scale is expanded. It leads to the growth of the upper bound on m_h as the number of $5 + \bar{5}$

supermultiplets rises. The investigations performed in [36] showed that the introduction of four or five $5+\bar{5}$ supermultiplets raise the theoretical bound on the lightest Higgs boson mass up to 155 GeV. If more than five multiplets are introduced at the SUSY breaking scale then the gauge couplings blow up before the Grand Unification scale and perturbation theory is not valid at $q^2 \sim M_X^2$.

Recently the upper bound on the lightest Higgs boson mass in more complicated SUSY models has been analyzed [44]–[47]. In particular, in addition to the gauge singlet superfield three $SU(2)$ triplets $\hat{T}_i$ with different hypercharges can be introduced into the Higgs boson superpotential:

$$\begin{aligned} W_{Higgs} &= \lambda\hat{Y}(\hat{H}_1\hat{H}_2)+\lambda_1(\hat{H}_1\hat{T}_0\hat{H}_2)+ \\ &\qquad +\chi_1(\hat{H}_1\hat{T}_1\hat{H}_1)+\chi_2(\hat{H}_2\hat{T}_1\hat{H}_2)+\ldots\,. \end{aligned} \tag{25}$$

As a result the expression for the upper bound on m_h changes

$$\begin{aligned} m_h^2 &\le M_Z^2\cos^2 2\beta+\Biggl[\left(\frac{\lambda^2}{2}+\frac{\lambda_1^2}{4}\right)\sin^2 2\beta+ \\ &\qquad +2\chi_1^2\cos^4\beta+2\chi_2^2\sin^4\beta\Biggr]v^2+\Delta\,. \end{aligned} \tag{26}$$

The appearance of triplet superfields destroys the gauge coupling constant unification at high energies. In order to restore the unification scheme of the electroweak and strong interactions one has to add several $SU(3)$ multiplets, for example four $(3+\bar{3})$, which do not participate in the $SU(2)\times U(1)$ interactions. A numerical analysis [45],[46] reveals that the unification of gauge couplings then occurs at the scale $\tilde{M}_X \sim 10^{17}$ GeV. In this case the upper bound on the lightest Higgs boson mass rises with growth of $\tan\beta$ and for $\tan\beta \gg 1$ reaches 190 GeV [46].

Also the upper bound on m_h is raised if in the MSSM a fourth generation of the quarks and leptons exists [47]. However up to now there has not been found any evidence of the existence of the fourth generation in the SM or MSSM. Moreover new particles give considerable contributions to the electroweak observables which upset the agreement between theoretical predictions and the results of experimental measurements. Thus the growth of the upper bound on the lightest Higgs boson mass in the supersymmetric models is usually accompanied by substantial increase in the number of particles that may be considered as the main drawback of these models.

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Table 1. Spectrum of the superparticles and Higgs bosons for $m_t(174\text{ GeV})=165$ GeV, $A = m_0 = 0$ and for different initial values of $h_t^2(0)$ and $\lambda^2(0)$ (all parameters and masses are given in GeV).

	$\mu_{eff} < 0$			$\mu_{eff} > 0$		
$\lambda^2(0)$	2	10	10	2	10	10
$h_t^2(0)$	10	10	2	10	10	2
$M_{1/2}$	-392.8	-392.8	-392.8	-392.8	-392.8	-392.8
$\tan\beta$	1.736	1.883	2.439	1.736	1.883	2.439
μ_{eff}	-771.4	-727.8	-641.8	772.4	728.6	642.3
B_0	622.5	1008.0	886.2	-988.1	-1629.1	-1583.3
y	-0.0014	-0.0015	-0.0012	-0.0003	-0.0004	-0.0005
$\mu'(t_0)$	1693.9	1671.5	1749.8	-1941.4	-1899.8	-1943.1
$\mathbf{m_h(t_0)}$ **(1–loop)**	**123.6**	**134.1**	**137.6**	**112.4**	**125.0**	**131.2**
$\mathbf{m_h(t_0)}$ **(2–loop)**	**113.0**	**124.4**	**127.8**	**105.5**	**118.4**	**123.6**
$M_3(1\,\text{TeV})$	1000	1000	1000	1000	1000	1000
$m_{\tilde{t}_1}(1\,\text{TeV})$	891.6	890.2	890.5	837.0	840.6	853.5
$m_{\tilde{t}_2}(1\,\text{TeV})$	622.2	630.3	648.5	693.8	695.1	696.4
$m_H(1\,\text{TeV})$	961.0	896.2	758.5	963.3	898.5	761.1
$m_S(1\,\text{TeV})$	1999.8	2147.4	2187.2	2405.3	2623.4	2663.8
$m_{A_1}(1\,\text{TeV})$	1374.8	1123.2	1294.0	1390.6	953.9	965.1
$m_{A_2}(1\,\text{TeV})$	949.8	857.6	735.6	951.6	704.3	674.3
$m_{\tilde{\chi}_1}(t_0)$	160.1	160.0	159.9	164.6	164.6	164.4
$m_{\tilde{\chi}_2}(t_0)$	311.9	311.1	309.4	328.1	327.8	326.4
$\lvert m_{\tilde{\chi}_3}(1\,\text{TeV})\rvert$	795.8	753.7	665.8	797.2	755.1	668.1
$\lvert m_{\tilde{\chi}_4}(1\,\text{TeV})\rvert$	807.8	764.7	677.1	800.9	755.9	666.7
$\lvert m_{\tilde{\chi}_5}(1\,\text{TeV})\rvert$	1711.2	1700.7	1790.0	1960.7	1931.8	1986.5
$m_{\tilde{\chi}_1^\pm}(t_0)$	311.6	310.7	309.0	328.1	327.8	326.4
$m_{\tilde{\chi}_2^\pm}(1\,\text{TeV})$	806.0	763.3	676.7	800.4	757.0	669.0

Figure captions

Figure 1. The particle spectrum in the modified NMSSM as a function of $z = \mu'/1\,\mathrm{TeV}$ and $x = A/M_{1/2}$ for $h_t^2(0) = \lambda^2(0) = 10$, $m_0^2 = 0$, $M_3 = 1\,\mathrm{TeV}$ and $\mu_{eff} \le 0$. Thick and thin curves in Fig.1a and 1b correspond to the lightest higgs boson mass calculated in the one– and two–loop approximation respectively. In Fig.1c and 1d thick and thin curves reproduce the dependence of CP–even Higgs boson masses m_S and m_H on z and x while dotted and dashed curves represent the CP–odd Higgs boson masses m_{A_1} and m_{A_2} as a functions of these parameters. The dashed–dotted curve in Fig.1c corresponds to the mass of the heaviest neutralino.

Figure 2. The particle spectrum in the modified NMSSM as a function of $z = \mu'/1\,\mathrm{TeV}$ and $x = A/M_{1/2}$ for $h_t^2(0) = \lambda^2(0) = 10$, $m_0^2 = 0$, $M_3 = 1\,\mathrm{TeV}$ and $\mu_{eff} \ge 0$. The notations are the same as in Fig.1.

Figure 3. Upper bound on the lightest Higgs boson mass in the MSSM (lower curve) and in the modified NMSSM (upper curve) as a function of $\tan\beta$ for $M_3 = 2\,\mathrm{TeV}$.

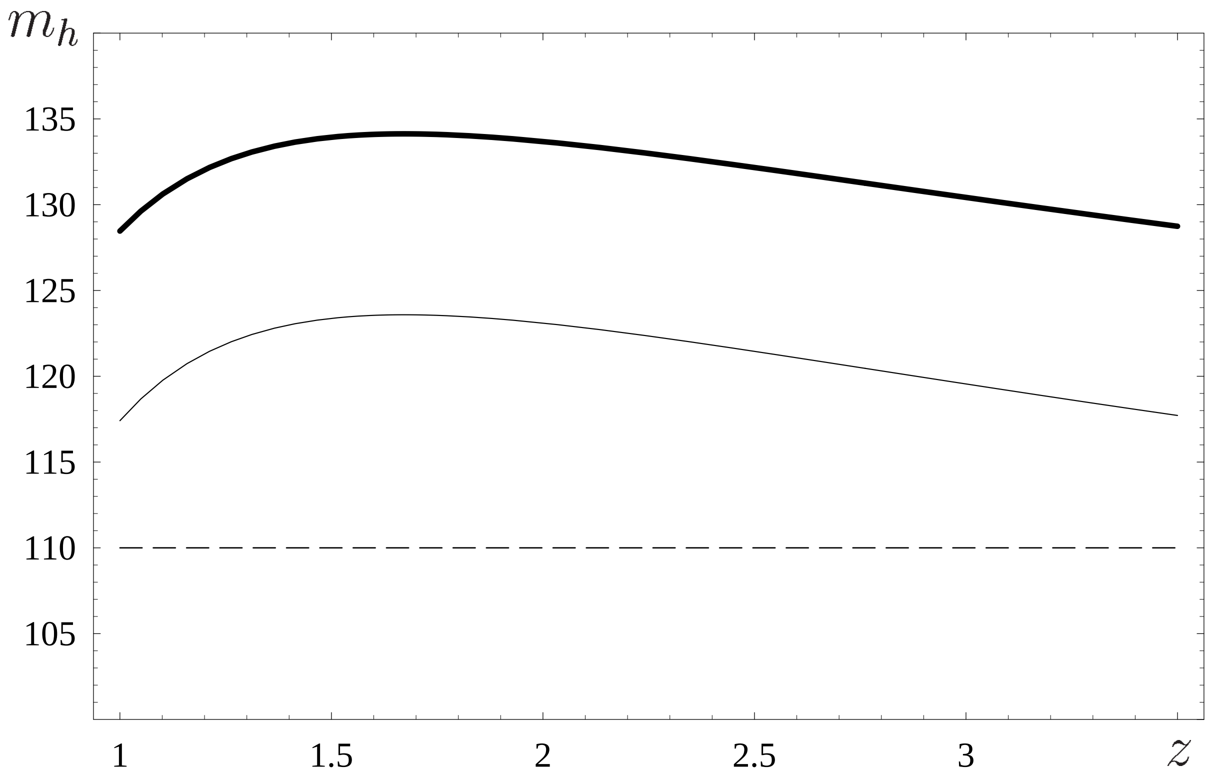

m_h
135
130
125
120
115
110
105
1
1.5
2
2.5
3
z

Fig.1a.

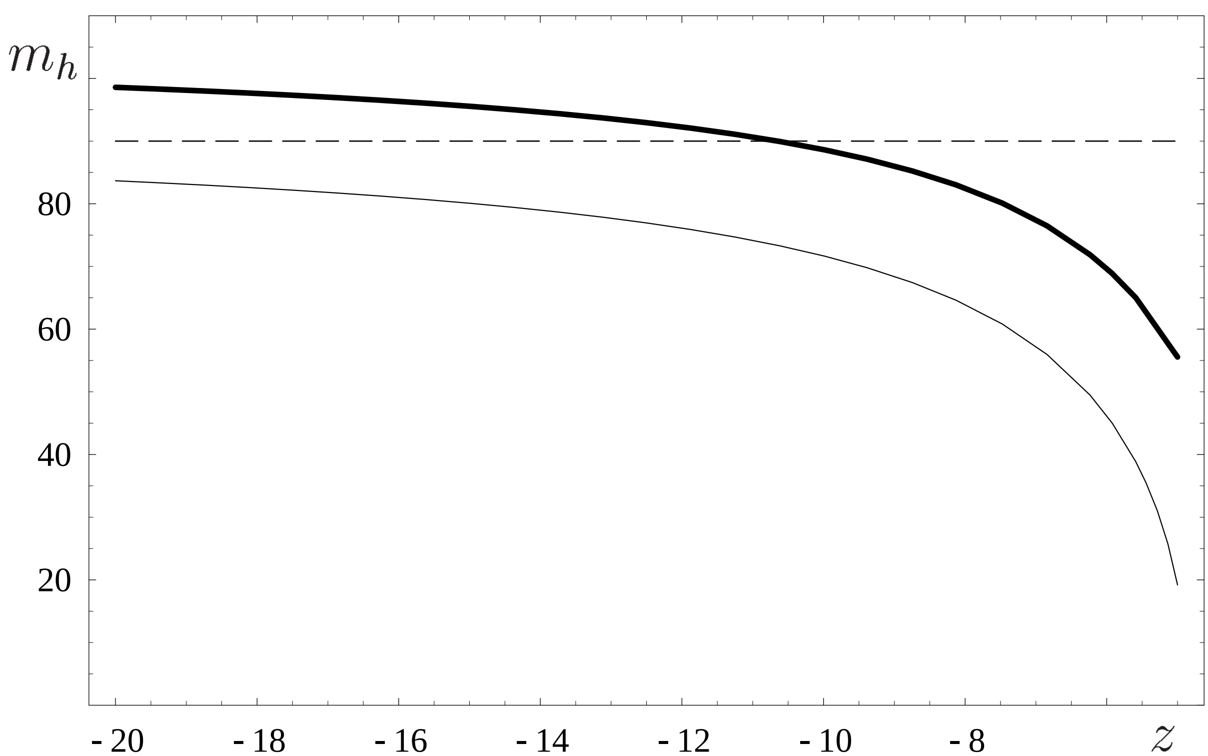

m_h
80
60
40
20
- 20
- 18
- 16
- 14
- 12
- 10
- 8
z

Fig.1b.

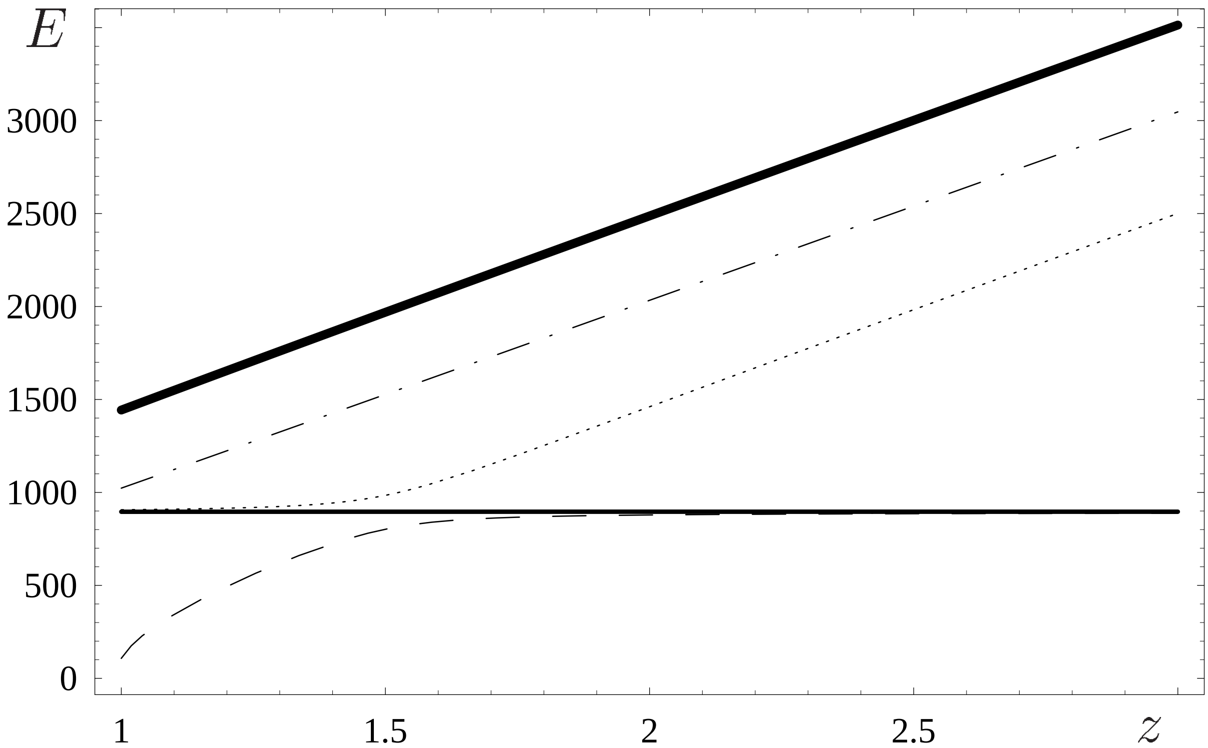

E
3000
2500
2000
1500
1000
500
0
1
1.5
2
2.5
z

Fig.1c.

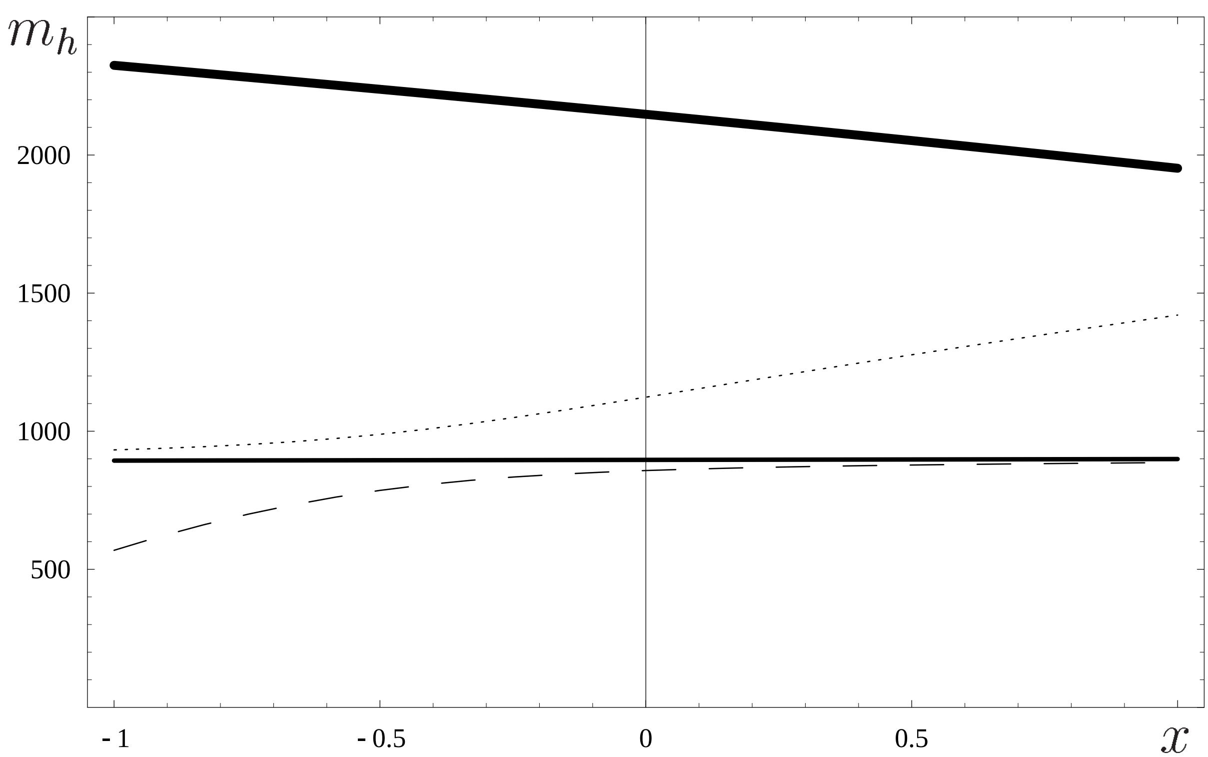

m_h
2000
1500
1000
500
-1
-0.5
0
0.5
x

Fig.1d.

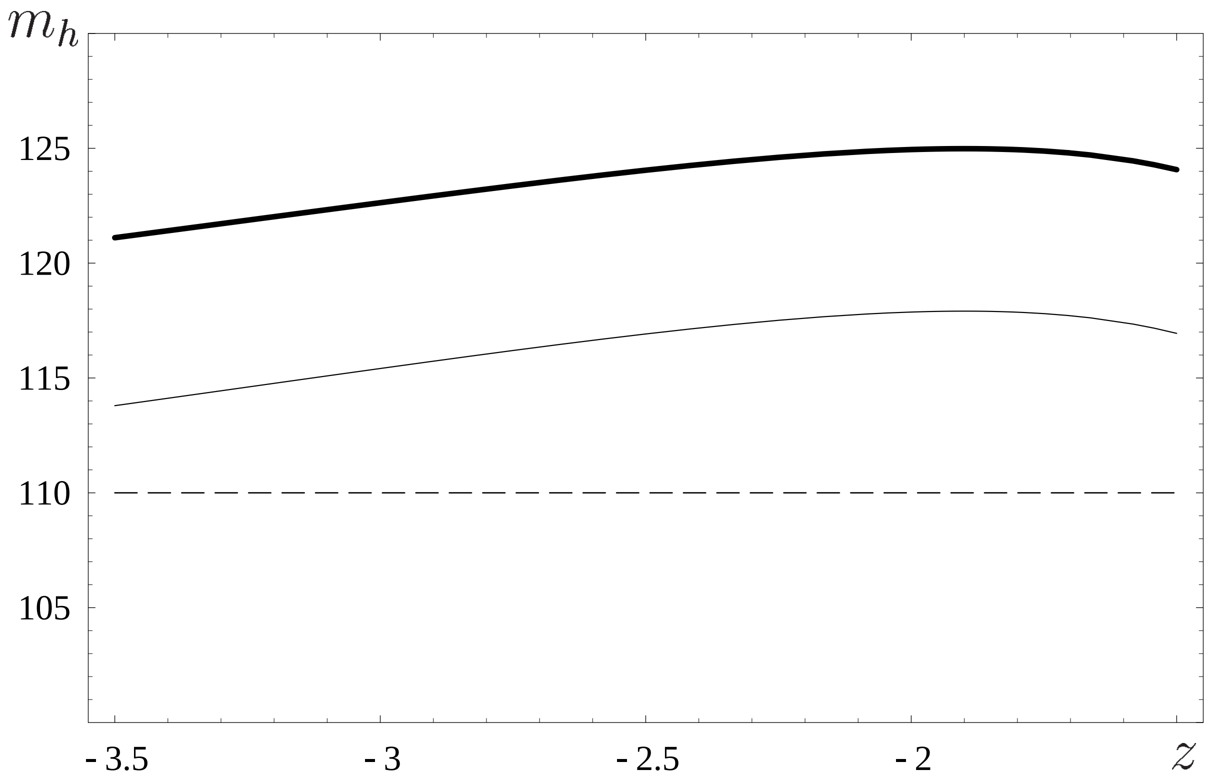

m_h
125
120
115
110
105
- 3.5
- 3
- 2.5
- 2
z

Fig.2a.

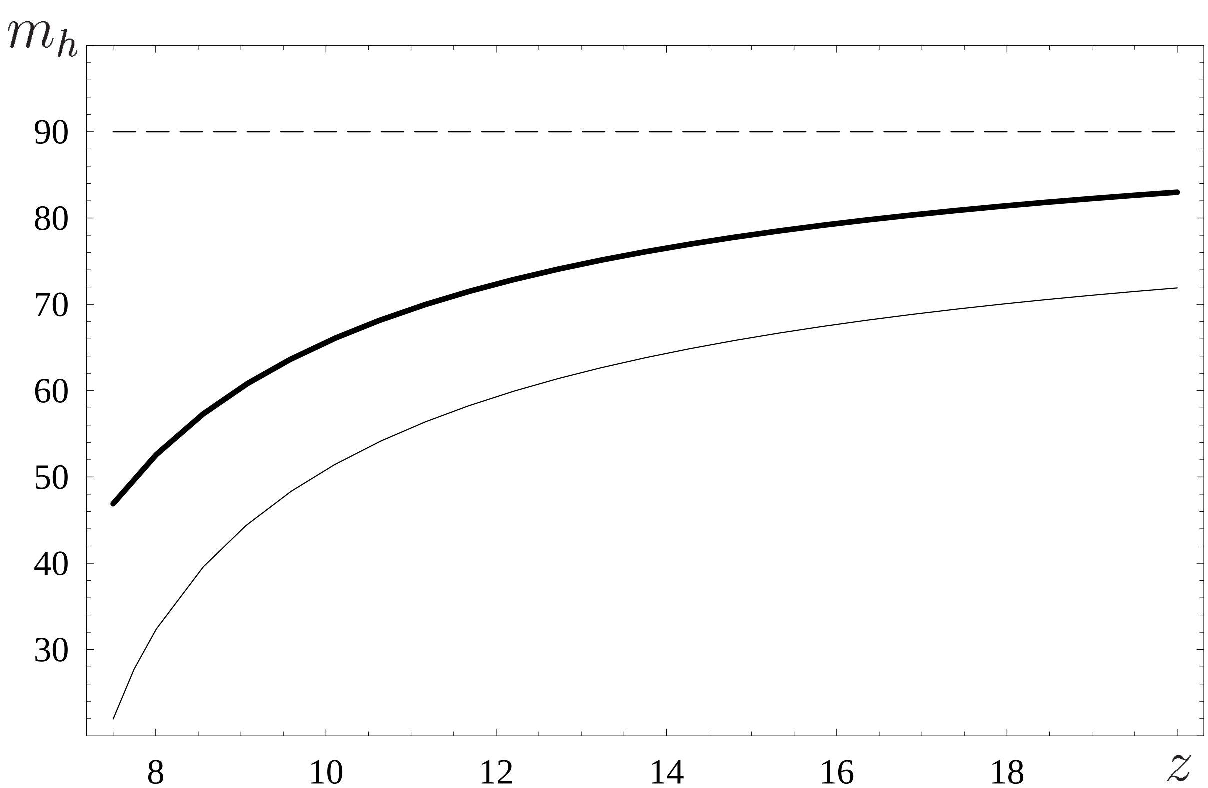
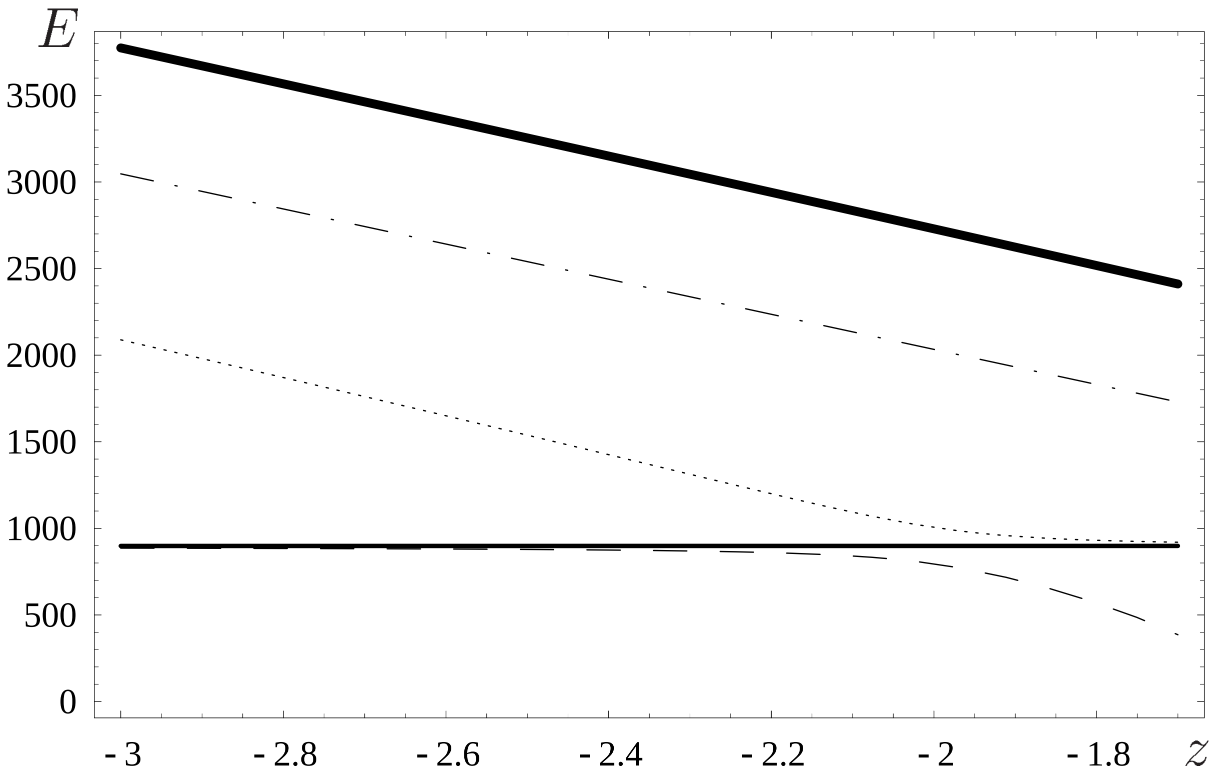
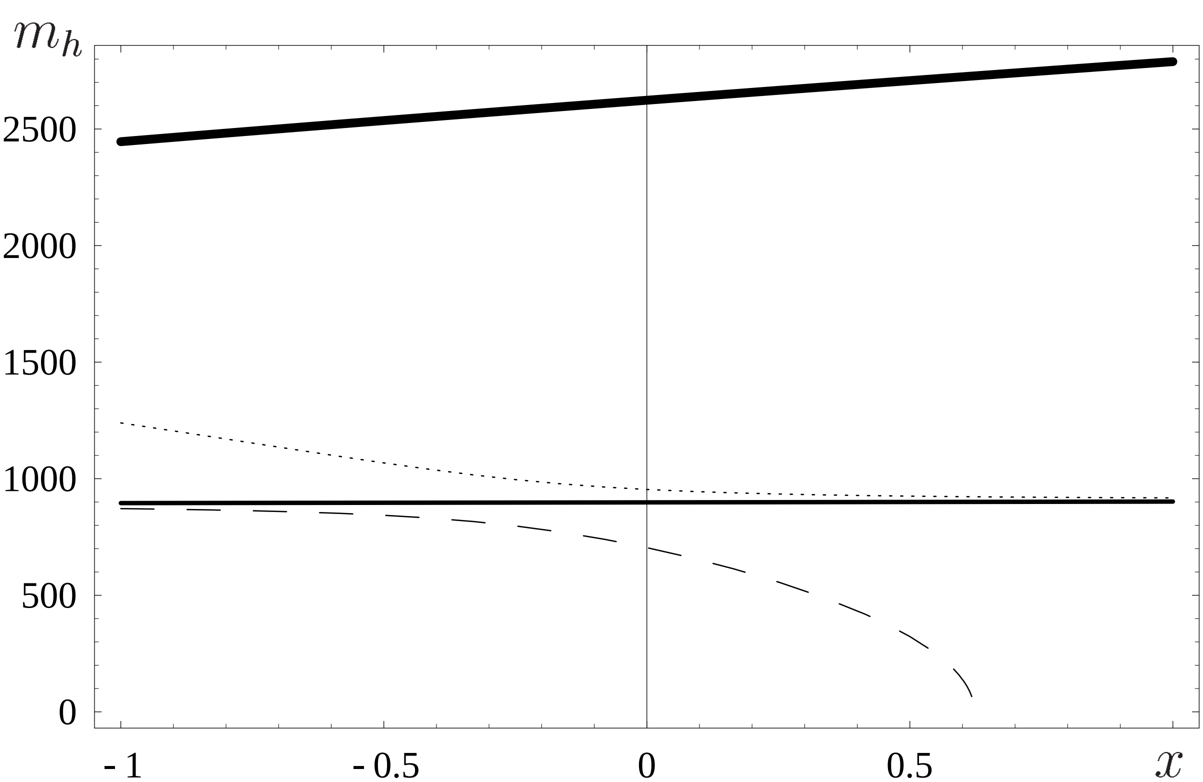

m_h
90
80
70
60
50
40
30
8
10
12
14
16
18
z

Fig.2b.



E
3500
3000
2500
2000
1500
1000
500
0
- 3
- 2.8
- 2.6
- 2.4
- 2.2
- 2
- 1.8
z

Fig.2c.



m_h
2500
2000
1500
1000
500
0
- 1
- 0.5
0
0.5
x

Fig.2d.

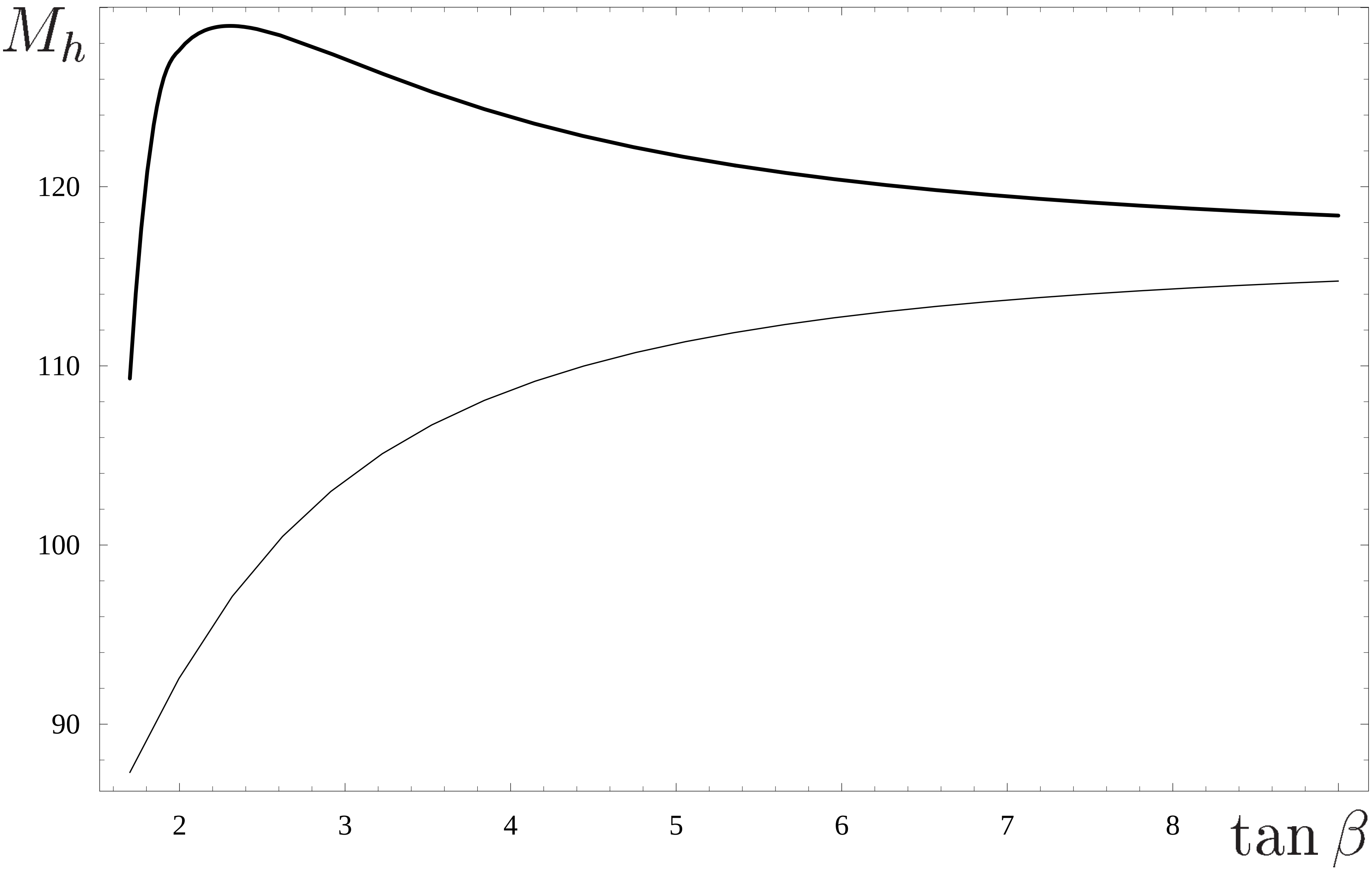


Fig.3.